IMPACT OF ROOF INSULATION THICKNESS ON LIFE-CYCLE COSTING OF BUILT-UP ROOF SYSTEMS

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Skyrocketing energy costs, projected far into the foreseeable future, have pushed the long-neglected technique of life-cycle costing to the forefront of building economic considerations. The building industry has traditionally focused on first-cost economy, oblivious of longterm economy. But the economic pain of soaring heating and cooling bills for inadequately insulated buildings is now to great to be ignored. For years, thermal insulation has been the best general longterm investment of any building component, typically repaying its investment within two to five years. Today, the pay back period is even shorter. Builtup roof systems formerly insulated with 1 in. of mediocre insulation are today getting 2 or 3 inches of higher grade insulation.

Thickened, higher grade roof insulation, however, raises fears of shortened roof life. Some industry spokesman consequently question its value much beyond the typical 1 in. thickness traditionally specified. Added insulation thickness, they admit, reduces a building's annual operating cost. But suppose this added insulation significantly shortens the membrane's service life, requiring an expensive tearoff reroofing job years earlier than required for a more lightly insulated roof. The increased capital cost might nullify, or even outweigh, the heating and cooling energy savings. Estimates of the various costs and service lives are indispensable to a resolution of this dilemma. As this paper will demonstrate, these estimates lead to this general conclusion:

A builtup roof system with optimum insulation thickness will generally prove more economical on a longterm (life-cycle) cost basis than an insulated builtup roof system with a traditional 1-in. insulation thickness.

LIFE-SHORTENING EFFECTS OF THERMAL INSULATION

Before launching into life-cycle costing economics, consider first the deleterious effects of thickened thermal insulation. Thickened, more thermally resistant insulation exposes a builtup membrane to several specific life-shortening effects. Roofing experts have cited the following:

- Accelerated chemical degradation of bitumen
- Increased splitting hazard
- Reduced impact resistance
- Increased risk of slippage[1]

Most of these hazards result from the greater temperature range—hotter in summer, colder in winter—experienced by a heavily insulated membrane. This phenomenon can be explained by thermodynamic principles. A stable, "equilibrium" temperature occurs whenever the total energy entering the roof membrane (or any other body) equals the total energy leaving it. Consider, first, the thermal situation on a hot, sunny day, with the roof exposed to a steady level of solar radiation. This radiated solar energy is rejected through four physical mechanisms: reflection, convection, conduction, and re-radiation of absorbed energy. Thickened insulation retards conduction. It has no effect on reflection, which depends on surface color, and very little effect on convection, which depends chiefly on wind speed. That leaves increased re-radiation to offset most of the reduced conduction loss produced by the thickened insulation, and increased re-radiation requires a higher surface temperature (see Fig. 1). (In accordance with the famous Stefan-Holtzmann equation, the energy rejected by re-radiation is proportional to the fourth power of the absolute temperature.)

On a clear, cold night, when there are no clouds to absorb and re-radiate heat energy radiated from the earth, the surface temperature of a heavily insulated membrane will drop slightly below that of a lightly insulated membrane, because the energy radiated by the membrane into space will not be replenished as fast by conduction from the warmer building interior. Thermal equilibrium will consequently be established at a lower membrane temperature.

Chemical degradation of the bitumen will accelerate with the higher summer surface temperature from thickened insulation. The rate of oxidation, chief agent of this chemical degradation, rises exponentially with temperature, its rate possibly doubling for each 18°F temperature rise.[2] Thus arises the fear that even a relatively small rise in temperature might significantly shorten membrane life, because the waterproofing quality of the builtup membrane depends on the bitumen's durability.

Increased splitting hazard results from several factors associated with thickened insulation. Most important is thickened insulation's reduced horizontal shearing resistance to membrane contraction, which could be produced either by temperature drop or by drying of the felts. Horizontal shearing resistance can be assumed inversely proportional to insulation thickness—i.e., doubled insulation thickness reduces horizontal shearing resistance by half. A minor factor to increase the membrane splitting hazard is the previously discussed, slightly lower membrane temperature that would occur over heavy insulation on cold, clear nights.

Reduced impact resistance makes a roof vulnerable to roof traffic and hailstone damage. Since impact resistance is roughly a function of compressive shortening under dynamic loading, it, too, is inversely proportional to insulation thickness.

Increased risk of slippage from increased thermal insulation is reported by Rissmiller, senior research associate at Jim Walter Research Corporation. In lightly insulated roof systems, slippage seldom occurs on roofs with less than 1 in. slope, reports Rissmiller. But today, it is not unusual to encounter slippage on roofs with 2-3 inches of insulation and only 1/2 in. slope.[3]

ASSESSMENT OF THE HAZARDS

The foregoing catalog of hazards makes the problems of thickened insulation appear substantial. But analysis of the magnitudes of anticipated temperature changes associated with most of these hazards alleviates the concern. Several studies have indicated that roof surface temperature rises only a few degrees even when insulation is drastically thickened. Rossiter-Mathey calculate a maximum temperature increase of 4°F, from 153°F for a black-surfaced roof with 0.25 U factor (1-in. fiberboard insulation) to 157°F with a .066 U factor (5-in. fiberboard insulation).[4]

Cash and Gumpertz report similar results, a 6°F higher membrane temperature change for a 100°F air temperature change for a system with 4 inches of fiberboard insulation compared with a system having only 1-in. fiberboard.[5]

I have made a calculation eliminating all convective heat loss, a calculation that pushes the surface temperature rise somewhat past its theoretical limit. Yet this calculation indicates only a 9°F temperature difference for a change from 1 to 5 in. of fiberboard (or perlite board) insulation (see Fig. 2). Since this calculation assumes that all additional heat losses are effected solely through re-radiation, thus requiring **maximum** surface temperature rise, it is obviously far too conservative to be realistic. The true value obviously lies closer to the Rossiter-Mathey figure of 4°F.

The Rossiter-Mathey study indicates a further significant fact about the effect of insulation on roof temperature, a fact borne out by earlier empirical field research by W. C. Cullen. The difference in roof membrane temperature between builtup roof systems with no insulation and just 1/2 in. insulation is greater than the difference between builtup roof systems with 1/2 in. and 5 in. insulation, according to the Rossiter-Mathey report.[6] Back in 1963, Cullen published experimental data indicating the big membrane temperature rise attributable to insulation. Cullen tested two black, smooth-surfaced roof samples on concrete decks, one totally uninsulated, the other with 2 in. of cork. Constantly recorded time-temperature curves over a two-year period revealed a 39°F maximum summer differential in membrane temperature (162°F for the insulated sample, 123°F for the uninsulated sample).[7] This 39°F temperature difference is much greater than the 11°F temperature difference computed by Rossiter-Mathey between uninsulated and 2-in. fiberboard insulation over steel deck. For my comparable calculation, again omitting convective heat loss, I compute a 28°F temperature difference (188°F vs. 160°F) for a black-surfaced steel deck roof assembly with 2 in. of fiberboard vs. an uninsulated membrane (Fig. 3). That gives fair agreement with the Cullen-Appleton field-recorded temperature difference of 39°F, which it must be remembered, is the extreme temperature difference occuring over a two-year period.

In any event, no researcher has failed to note the greater temperature difference between uninsulated and lightly insulated membranes than that between lightly and heavily insulated membranes. Moreover, no one argues for omitting all insulation to lengthen membrane service life. Thus the problem, precisely defined, is to determine the point at which the membrane life-shortening effects of thickened insulation outweight the energy savings minus the incremental cost of thickened insulation.

Note, further, that roof color apparently has a significantly greater effect on membrane temperature than insulation thickness. For roofs of equal insulation thickness, the calculated temperature difference between a black and gray surface is 15°F; between black and white, it is 27°F.[8]

Yet this color effect on membrane temperature is often ignored, presumably because of the added first cost and maintenance cost of replenishing the reflective coating. Compared with surface color, insulation thickness is apparently a minor factor promoting chemical degradation of the bitumen.

Splitting, impact, and slippage hazards suggest an investigation into the structural properties of the specified insulation, and the superiority of thinner, more efficient composite insulation boards over thickened layers of less efficient insulations. Mechanical fastening of the insulation can reduce splitting risks. Specification of an extra ply of felt, possibly two extra plies, can provide additional tensile strength to resist contraction stress. Closer spacing of expansion/contraction joints is still another means of reducing splitting hazards. And mechanical anchorage of the base sheet can similarly reduce the slippage risk.

In summary, none of the hazards accompanying thickened, more thermally efficient insulation appear to impose severe difficulties. They all appear to be remediable at slighly increased first cost, measured in minor fractions, not multiples of the basic builtup roof system cost.

APPROACH TO THE PROBLEM

There are two approaches to the hazards posed by increased roof insulation thickness:

(a) You can design a stronger, more expensive roof membrane (or system) designed to maintain 20-year service life.

(b) You can simply specify the same membrane specified for conventional insulation thickness and accept a shortened service life.

In the normal life-cycle approach, you would make quantitive assumptions about (a) or (b) and then run through calculations to see which produced the lowest life-cycle (i.e., longterm) owning cost. Since the assumptions involved in (a), and especially in (b) are so speculative at this stage of roofing technology, I shall turn the problem around. Instead of comparing two systems of equal (or unequal) service life for total owning cost, I shall estimate the annual Operating and Maintenance (O & M) savings attributable to optimal (or close to optimal) insulation thickness. I shall then calculate the required service for that system to equal the longterm economy of a conventionally insulated roof system. I shall also calculate the O & M savings available for strengthening the membrane. We can then judge whether (a) the additional funds available for strengthening the membrane can **reasonably** be expected to lengthen its service life to 20 years, **or** (b) whether the same membrane specified for conventional insulation thickness can be expected to last long enough to reduce the heavily insulated roof system's life-cycle cost to the 20-year life-cycle cost of the lightly insulated roof system.

LIFE-CYCLE COSTING ASSUMPTIONS

For the assumed problem, consider an oil-heated school, with steel deck roof structure, no ceiling, current heating oil cost = 45 cents/gal., overall heating system efficiency = 60%, 5,600-deg-day climate (Morristown, NJ). Consider heat cost savings only. As a further concession to conservative fuel-saving estimates, reduce the number of full heating days by 35%, to allow for holidays and weekends, when heat can be turned down to 55°F. Use 55°F instead of 65°F as the degree-day base, to allow for heat load of human occupants and lighting. Since there are roughly 210 days in an annual season, the reduced deg-days are computed as follows:

$$\text{Design deg-days} = 5{,}600 - 10 \times 210 = 3{,}500$$

Further assumptions: 6% interest rate for Present-Worth cost calculations, 10% annual fuel cost escalation (vs. 15% or more over the past five years).

We first calculate optimum insulation thickness for 20-year service life on a Present-Worth basis. (It is necessary to reduce future costs to a Present-Worth basis to account for the interest value of money, since a dollar saved today is worth considerably more than a dollar saved 10 years from now.) The only variables assumed relevant to optimum insulation thickness are roof insulation thickness and the resulting fuel cost differences. Thus optimum insulation for an assumed 20-year service life minimizes total cost. Total cost comprises two components:

(1) Insulation cost, and

(2) Heating fuel cost, totaled for 20 years on a Present-Worth basis.

As the next step, calculate insulation cost, C_i, and Fuel cost, C_f. Roof insulation generally sells at a fairly constant price per unit of thermal resistance, R. Since R factor is measured dry, a condition rarely satisfied in the field, reduce the published R factor by an average 10%. (The designer should, of course, note the varying thermal effects of moisture on the specific insulation.) Under these conservative assumptions, we can use a unit cost, $C_i = \$.085\ R/ft^2$, a figure that should fall well within the overwhelming majority of price quotes, especially on large projects.

Fuel cost, C_f, is calculated from the basic formula:

$C_f\ (\$/ft^2)$ = Annual heating load (Btu/ft^2) x Present-Worth sum of 20 annual energy bills.

Optimum insulation works out to $R_i = 11.4$, or roughly 3-1/4 in. of insulation with an effective k factor of 0.275. (See Fig. 4 for optimum insulation calculation.) Compared with a conventionally insulated roof (1-in.-thick insulation) an optimally insulated roof system would save $1.05 psf over a 20-year service life (see Fig. 4). You could use this saving for one of two purposes:

(1) to finance membrane-strengthening measures (e.g., an extra ply of felt, additional expansion-contraction joints) to improve membrane durability, or
(2) to finance tearoff-reroofing of a conventional membrane suffering shortened life because of increased stresses resulting from thickened insulation.

If we spend the $1.05 saving for membrane-strenghening improvements, we could add an extra felt ply, for example, for about $0.10 psf, two additional plies for $0.20 psf. For similarly slight costs, reckoned in pennies psf, we could mechanically anchor the insulation and install additional, more closely spaced expansion/contraction joints, two measures designed to reduce splitting hazards aggravated by thickened insulation. Additional investment in heat-reflective aggregate, or for periodic renewal of a smooth-surfaced roof's reflective coating, should more than counter the slight increase in roof surface temperature attributable to thickened insulation. This $1.05-psf saving gives us nearly twice as much money to spend on our heavily insulated membrane. Some of this money should go for tightening field inspecting. Beyond reasonable doubt, such a system should outlast a conventionally designed and built membrane over light insulation.

Now consider the second approach, using the savings to finance earlier tearoff-replacement. It turns out that the required service life, for equivalent life-cycle cost for the heavily insulated membrane is less than 15 years, against an assumed 20 years for the conventional, lightly insulated roof system (see Fig. 6 for computation and assumptions for this computation). In other words, if the hazards of thickened insulation shorten the membrane service life by five years or less then thickened insulation provides more longterm economy. It appears reasonable to assume that thickened insulation will not shorten membrane life by more than one-quarter its normal service life over light insulation.

Actually, the best approach is doubtless partway between the two extreme approaches of applying all or none of the $1.05 saving to strenghening the membrane. For an expenditure of less than half that amount, we should be able to lengthen the heavily insulated membrane's service life to that of a conventional lightly insulated membrane. And we should be able to pocket to $0.60 to $0.80 psf as a longterm saving. Moreover, because of ultraconservative assumptions made in this example (e.g., ignoring cooling energy savings), actual savings should probably exceed $1.00 psf.

Optimum insulation thickness will obviously vary with local climatic conditions and financial conditions (interest rates, tax rates, energy cost escalation rates, etc.). But if in doubt, there are sound reasons for favoring thick insulation over the usual thin insulation. This advice contradicts the conventional wisdom, which favors the status quo unless there are overwhelmingly convincing reasons to change. But if the arguments favoring thicker insulation are tenuous, the arguments for light insulation are even more so. In an era of projected longterm energy shortages and rapidly escalating energy costs far into the foreseeable future, the safer gamble is to favor energy savings even at some slight threat to membrane service life. And the greater attention paid to this threat might constitute precisely the added effort required to avert premature membrane failure.

ACKNOWLEDGEMENTS

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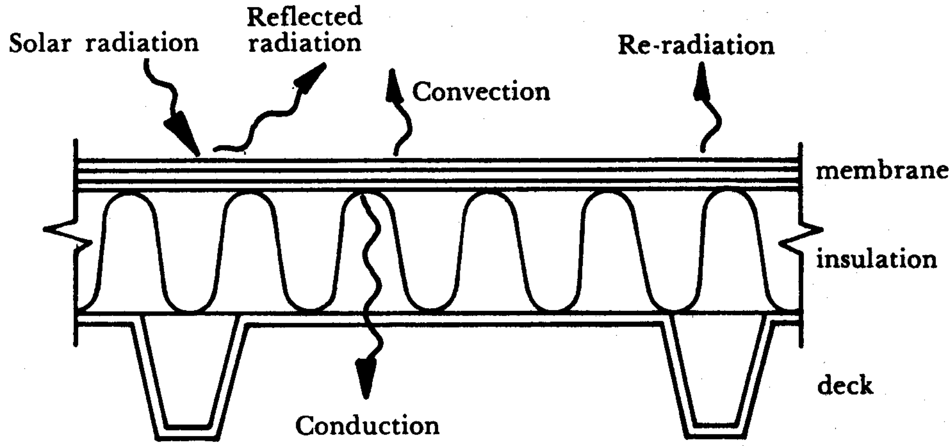
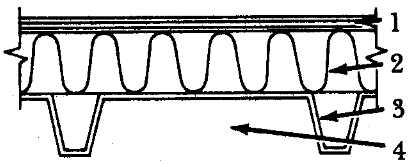


Fig. 1. Under thermal equilibrium, absorbed solar radiation (Solar radiation – reflected radiation) is balanced by conduction, convection, and re-radiation heat losses. Thickened insulation increases roof surface temperature by reducing conduction loss, thus requiring increased heat rejection by convection and re-radiation, both of which increase at higher temperature. (Reflective heat loss is color-dependent, varying only with solar radiation intensity.)



Component	R
1. BUR membrane	0.33
2. Insulation	R_i
3. Steel deck	0
4. Interior air film	0.92
(downward heat flow)	$R_t = 1.25 + R_i$

Ignore reflected heat loss (which remains constant) and convected heat loss (which varies with changing surface temperature). Then for thermal equalibrium,

Absorbed solar radiation = Conducted loss + Re-radiated loss

$\alpha I = U(T_r - T_i) + \sigma(T_r^4 - T_s^4)$, in which

α = Black roof's surface absorptivity = 0.90

I = Solar-radiation intensity on flat roof surface = 300 Btu/hr. (See Rossiter-Mathey paper)

T_i = Inside temperature at ceiling level = 80°F (540°R)

T_s = Sky temperature = 50°F (410°R)

T_r = Membrane surface temperature

σ = Stefan-Boltzmann constant - 0.1713×10^{-8}

For 5-in. Thick insulation (k = 0.36) $U = \dfrac{1}{1.25 + (5 \times 2.78)} = .066$

$$0.9 \times 300 = .066\,(T_r - 540) + 0.1713 \times 10^{-8}\ (T^4 - 410^4)$$
$$270 = .066T_r - 35.6 + 0.1713 \times 10^{-8}\, T_r^4 - 48.4$$
$$0.1713 \times 10^{-8}\, T_r^4 + .066T_r - 354 = 0$$
$$T_R = 652°R$$
$$T_R = 192°\,F$$

For 1-in. Thick insulation, $U = \dfrac{1}{1.25 + 2.78} = 0.25$

$$270 = 0.25\,(T_r - 540) + 0.1713 \times 10^{-8}\ (T_r^4 - 410^4)$$
$$0.1713 \times 10^{-8}\, T_R^4 + 0.25T_r - 453 = 0$$
$$T_r = 643°R$$
$$= 183°F$$

ΔT_R = 192-183 = 9°F surface temperature difference between 5 in. and 1 in. insulation thickness.

Fig. 2. The above computation, omitting convective heat loss, indicates a 9°F surface temperature differential for black-surfaced membranes insulated with 5 inches vs. 1 inch of fiberboard insulation.

Calculate the temperature difference for black surfaced membranes insulated with 2 inches of fiberboard and uninsulated. (See Fig. 2 for builtup roof system components and other problem data.)

For 2-in. fiberboard insulation

$$U = \frac{1}{1.25 + 5.76} = 0.14$$

$$\alpha I = U(T_r - T_i) + \sigma(T_r^4 - T_s^4)$$

$$0.9 \times 300 = 0.14(T_r - 540) + 0.1713 \times 10^{-8}(T_r^4 - 410^4)$$

$$0.1713 \times 10^{-8} T_r^4 + 0.14T_r - 394 = 0$$

$$T_r = 648°R$$

$$= 188°F$$

For uninsulated membrane

$$U = \frac{1}{1.25} = 0.80$$

$$0.9 \times 300 = 0.8(T_r - 540) + 0.1713 \times 10^{-8}(T_r^4 - 410^4)$$

$$270 = 0.8T_r - 432 + 0.1713 \times 10^{-8} T_r^4 - 48$$

$$0.1713 \times 10^{-8} T_r^4 + 0.8T_r - 750 = 0$$

$$T_r = 620°R$$

$$= 160°F$$

$$\Delta T = 188°F - 160°F$$

$$= 28°F$$

Fig. 3. Above computation, again omitting convective heat loss, indicates a 28°F temperature differential for black-surfaced membranes insulated with 2 inches of fiberboard vs. no insulation.

$C_t = C_i + C_f$

in which C_t = Total heating cost per sq. ft. of roof area over 20 years ($\$/ft^2$)

C_i = Insulation cost = $.085R_i$ ($\$/ft^2$) ($R_i$ = Insulation's R factor)

C_f = Total 20-year heating cost = Annual heating load (Btu/ft^2) x Present-Worth Sum of 20 annual energy costs ($/Btu)

Annual heating load = U x Effective deg-days x 24 hours/day

$$= \frac{3500 \times 24}{1.25 + R_i}$$

Present-Worth of 20 yr. fuel cost $= F\,\frac{a(a^n-1)}{a-1}$, in which

$$F = \text{Current (year 0) fuel cost} = \frac{\$0.45/\text{gal. oil}}{140{,}000\ \text{Btu/gal} \times 0.60\ \text{(system efficiency)}}\ (\$/\text{Btu})$$

$a = \dfrac{1+f}{1+i}$, in which

f = annual fuel cost escalation rate (10%)

i = annual interest rate (6%)

n = No. of yrs (20)

$$a = \frac{1+0.10}{1+0.06} = 1.03774$$

$$\frac{a(a^n-1)}{a-1} = \frac{1.03774\,(1.03774^{20}-1)}{.03774}$$

$= 30.18$

$$C_f = \frac{3500 \times 24}{1.25 + R_i} \times \frac{0.45 \times 30.18}{140{,}000 \times 0.6} = \frac{13.58}{1.25 + R_i}$$

$$C_t = .085R_i + \frac{13.58}{1.25 + R_i}$$

For optimum R_i, C_t is a minimum and $\dfrac{dC_t}{dR_i} = 0$

$$\frac{d\left[\dfrac{a}{b + ck}\right]}{dx} = \frac{-ac}{b^2 + 2bextc^2x^2}$$

$$= \frac{-13.58}{1.25^2 + (2 \times 1.25)R_i + R_i^2}$$

$$\frac{d\left[.085R_i + \dfrac{13.58}{1.25 + R_i}\right]}{dR_i} = 0 = .085 + \frac{d\left[\dfrac{13.58}{1.25 + R_i}\right]}{dR_i}$$

$$0 = .085 - \frac{13.58}{1.56 + 2.5R_i + R_i^2}$$

$R_i^2 + 2.5R_i - 158.2 = 0$

$R_i = 11.4$

Assume k = 0.275 (10% reduction for moisture)

Req'd. thickness = kR = 0.275 x 11.4 = 3.13, say 3-1/4 in., R = 11.8

$$\text{Min } C_t = (.085 \times 11.8) + \frac{13.58}{1.25 + 11.8} = \$2.04$$

For 1-in. insulation (R_i = 3.64) $\qquad C_t = (.085 \times 3.64) + \dfrac{13.58}{1.25 + 3.64} = \$3.09$

TOTAL SAVING (3-1/4 vs. 1-in. insulation)

= \$3.09 − \$2.04

= \$1.05/ft²

FIGURE 4
OPTIMUM INSULATION THICKNESS

ECONOMICAL ROOF INSULATION THICKNESS

$$\text{Total Cost } (\$/ft^2) = \text{Insulation Cost} + \text{Energy Cost}$$

$$= .085R_i + \frac{13.58}{1.25 + R_i}$$

For $k = 0.275$, R per in. thickness = 3.64

$$\text{Total Cost } (\$/ft^2) = 0.31t + \frac{13.58}{1.25 + 3.64t}$$

(t = Insulation thickness)

t (in.)	Insul. Cost	Energy Cost	Total Cost
½	0.16	4.42	4.58
1	0.31	2.78	3.09
2	0.62	1.59	2.21
3	0.93	1.12	2.05
4	1.24	0.86	2.10
6	1.86	0.70	2.56
8	2.48	0.45	2.93
10	3.10	0.37	3.47
12	3.72	0.30	4.02
14	4.34	0.24	4.58
16	4.96	0.22	5.14

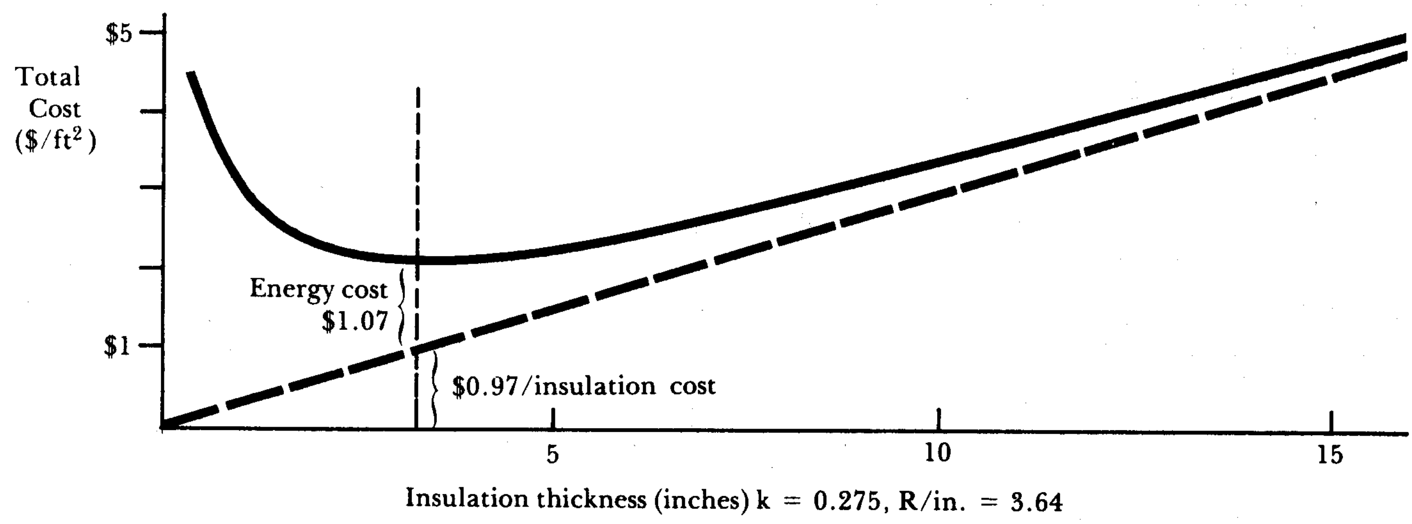


Fig. 5. Above graph shows how underinsulation raises total cost much more than a comparable degree of overinsulation. Also note that Total Cost is relatively insensitive to changes in insulation thickness at optimum thickness range.

Roof System A = Conventional BUR system with 1-in. (k = 0.275) insulation, with 20-year service life.
Roof System B = Heavily insulated BUR system with 3-1/4 in. insulation, service life = n yrs.
For equal life-cycle (longterm) cost over a total span of **2n** years, incremental costs for the two systems must be equal.
System A incremental cost = Additional energy costs + Tearoff-replacement cost (after 20 yrs)
System B incremental cost = Additional insulation cost + Tearoff-replacement cost (after **n** yrs)
Derive equations for these costs, on a Present-Worth basis

System A

$$\text{Additional energy cost (2n yrs)} = F\,\frac{a\,(a^{2n}-1)}{a-1}\text{, in which}$$

F = Additional energy cost in current year

$$F = \frac{\Delta U \times \overset{3500}{\text{deg.-days}} \times 24 \times 0.45}{140{,}000 \times 0.60} = \frac{\left(\frac{1}{4.89} - \frac{1}{13.05}\right) \times 3500 \times 24 \times 0.45}{140{,}000 \times 0.60\ \text{(efficiency)}} = \$.057/\text{ft}^2$$

$$\text{Additional energy cost } (\$/\text{ft}^2) = .057 \times \frac{1.038\,(1.038^{2n}-1)}{.038}$$

$$= 1.56 \times 1.038^{2n} - 1.56$$

For PW of Tearoff-replacement cost (after 20 yrs.), assume current cost = $\$2.30/\text{ft}^2$ with cost escalation rate = 8%, interest rate = 6%

$$\text{PW of tearoff-replacement} = 2.30\left(\frac{1.08}{1.06}\right)^{20} = \$3.34\text{psf}$$

For assumed roof life of 20 yrs. for **2n**-year time span, the proportionate cost for the tearoff-replacement is $\frac{2n-20}{20}$, for straight-line depreciation.
(Assumed service life = 20 yrs; actual roof service time = **2n** − 20 yrs.)

System B

Additional first cost for insulation = 0.31 x 2-1/4 in. = $0.70 psf
Additional cost for tearoff replacement at end of **n** yrs., at $3.00 psf for current cost, is equal to

$$\text{PW basis } \$3.00\left(\frac{1.08}{1.06}\right)^{n} = 3.1.019^{n}$$

Equate incremental costs for System A & System B.

System A		**System B**
Additional energy cost + Tearoff-replacement cost	=	Add'l insul. cost + Tearoff-replacement cost

$$1.56\text{x}1.038^{2n} - 1.56 + 3.34\,\frac{2n-20}{20} = 0.70 + 3\text{x}1.019^{n}$$

$$1.56\text{x}1.038^{2n} - 1.56 + 0.334n - 3.34 = 0.70 + 3\text{x}1.019^{n}$$

$$1.56\text{x}1.038^{2n} - 3\text{x}1.019 + 0.334 - 5.60 = 0$$

By trial-and-error, n = 14.7 years

Figure 6

CHECK WITH SIMPLIFIED METHOD FOR 20-YR TIME SPAN

For 20-year span, there will be no tearoff-replacement required for System A. Then the total savings (20-year energy savings - Additional insulation cost for System B) must finance tearoff-replacement cost for System B, with cost pro rated for (20-n) years to credit System B with additional service life. (System A's service life ends at 20th year.)

If tearoff-replacement cost escalates at 8%, cost at end of **n** years, equals

$\$3.00 \times 1.019^n$ (PW basis) (interest = 6%)

Depreciated on straight line

Tearoff-replacement cost $= 3 \times 1.019^n \times \frac{20-n}{n}$

This cost must equal the $1.05, 20-yr. saving (See Fig. 4)

$$1.05 = 3 \times 1.019^n \times \frac{20-n}{n}$$

$$3 \times 1.019^n (20-n) - 1.05n = 0$$

$$60 \times 1.019^n - 3n \times 1.019^n - 1.05n = 0$$

$$(60-3n) \times 1.019^n - 1.05n = 0$$

$$n = 15.9 \text{ years}$$

Figure 6 (cont'd.)