ECONOMIC OPTIMIZATION OF ROOF INSULATION THERMAL RESISTANCE

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ABSTRACT

The demand to save energy for economic and fuel conservation has focused attention on roof insulation. Current methods used to select the most economical quantity of roof insulation are often impractical and inadequate, as they do not consider fuel and insulation costs. This paper discusses a different method for determining the most economical quantity of roof insulation based on geographical location, local fuel costs, insulation costs and economic factors. Calculations determine the optimum thermal resistance, R, of a roof insulation system. The value, R, then can be used to select an optimum insulation quantity, as R is related to the insulation type and thickness.

Most roofing systems on public and industrial buildings were installed when savings in fuel costs did not justify the cost of installed roof insulation. Low initial cost was the primary factor in the choice of insulation quantities. Often, no insulation was applied where a roofing system for weather protection could be applied directly to the building's structural deck. In view of current fuel costs and availability, existing insulation quantities often are inadequate, causing economic loss and wasteful use of fossil fuels.

Recently, the need for energy conservation has focused attention on roof insulation systems. Building owners now need to optimize the amount of roof insulation for long-term economic benefits. Both new construction as well as reroofing designs require calculations to determine optimum insulation values.

DESIGN PROBLEMS

The difficulties encountered in optimizing a roof insulation design problem are many. Several variables for which values cannot always be obtained are involved. A few of these are:

- hours of building use,
- internal heat gains from lighting and other equipment,
- ambient temperature conditions,
- radiational heating and cooling,
- wall and window losses,
- thermostat settings and sensitivity, and
- nature of building occupancy.

Accurate energy calculations are virtually impossible because of such variables. However, several empirical calculations which ignore most of these variables are available to aid the roof designer in selecting an insulation quantity.

STATE-OF-THE-ART DESIGN

The American Society of Heating, Refrigerating, and Air Conditioning Engineers (ASHRAE) has developed a maximum value for thermal conductance for roofs in their Standard 9075.[1] The ASHRAE standard, however, can pose difficulties because it requires values for heat conductance through the ceiling, roof deck, roof membrane, and associated air spaces including outside and inside air films. Generally these values are not uniform over a large building. Therefore, the point of maximum heat flow as required by Standard 90-75, may be difficult to locate and may poorly represent overall building conditions. Also, the ASHRAE standard does not consider fuel, insulation, and installation labor costs.

The National Roofing Contractors Association (NRCA) has published an energy manual[2] to aid roof designers. Although the manual is extremely helpful in the calculation of energy costs for various insulation systems, it does not provide a method to directly determine the optimal value for thermal resistance, R, which yields the optimum insulation thicknesss. The NRCA manual, like the ASHRAE Standard, also uses total-to-outside conductance values. The inaccuracies in this method were discussed above.

In view of these problems, this paper discusses an alternate method for determining the most economical insulation thickness. The discussion is based on previous work done by Carl G. Cash[3,4] and Werner H. Gumpertz.[4] However, due to rapidly changing energy costs, the economic assumptions are different.

ECONOMIC MODEL

Assumptions

The economic model is based on the following assumptions:

- Several of the variables listed above are disregarded as they will remain constant despite roof system changes.
- Only costs incurred by energy losses and gains through the roof, not through adjacent materials and air spaces, are used. All components other than the roof remain constant despite roof insulation changes.
- A life cycle of 20 years is assumed. This is the average life cycle of a properly applied built-up roof membrane.
- Steady-state heat flow is assumed using "degree-days," a measure which reduces varying daily ambient temperatures to a single value used in energy calculations.
- Reduced costs for smaller heating and cooling equipment is not considered. In reroofing designs existing equipment usually will remain despite insulation changes. For new buildings, cost reductions due to smaller equipment may be considered. However, as seen in Table 4,[10] the

dollars saved are not significant for most buildings. For very large roof areas, equipment savings may be incorporated into the calculations. Equipment savings will tend to increase the value for optimum thermal resistance (R).

- Added insulation may cause design changes due to an increase in roof system thickness. Often wood blocking for mechanical supports or attachments located at roof perimeters, expansion joints, and penetrations for pipes or HVAC equipment must be added when insulation height increases. These changes can cause severe increases in roof system installation costs. Such costs should be estimated by the roof designer and added to installation costs for the insulation. The effect of increasing installation costs is to lower the optimum R value.
- Economic factors are used to determine the effects of rising fuel costs and the value of money as determined by the discount rate used by a building owner for cost accounting. Total monetary values for the 20-year life cycle are converted to equivalent uniform annaul costs (EUAC).

Definitions of Variables

R = thermal resistance of insulation, (hrs × ft.^{2} × degree F)/Btu

Heating Calculations

P_h = fuel costs of heat lost through roof insulation, \$/(ft^2 × year)

F_h = cost of heating fuel in conventional units defined by type of fuel used for heating, \$/unit

B_h = heating value of fuel per unit, Btu/unit

E_h = efficiency of heating plant and distribution system, percent

H = heating degree-days, degree F × days/year

Cooling calculations

P_c = energy costs for coolign due to heat gain through roof, \$ (ft^2 × year)

F_c = cost of cooling energy in conventional units, \$/unit

B_c = cooling value of one unit, Btu/unit

E_c = efficiency of cooling plant and distribution system, percent

C = cooling degree-days, (degree F × days)/year

Insulation cost calculation

P_I = installed cost of insulation, \$/(ft^2 × year)

J = installation cost per unit R of insulation

Economic Factors

The values obtained for first-year heating and coolign cost using current values will increase due to rising fuel costs. Therefore, it is necessary to multiply current prices by an economic factor based on the yearly increase in fuel cost as a percentage and the discount rate used by the building owner. Fuel costs are assumed to rise exponentially over the 20-year life-cycle. The cash flow diagram for fuel costs in Figure 1 illustrates this. End-of-year cash flows are assumed.

A factor (A) converts the first-year fuel cost to a Present Worth total fuel cost for the 20-year life cycle.

In estimating the annual fuel cost increase, the designer first should consider past trends and future predictions. If the value is questionable, use a slightly higher percentage. This will guard against energy price booms which may occur, and also will better conserve fuel resources by increasing the optimum R value.

j = approximate yearly change in energy costs, percent

i = discount rate used by building owners, percent

All Present Worth cost values are converted to an equivalent uniform annual cost (EUAC) using a factor (B) dependent only on the change in energy cost. Changing Present Worth values to EUAC values does not affect the calculation for R, but since construction costs are usually distributed over many years, most building owners are more interested in yearly cash flows.

COST CALCULATIONS

Economic Factors are calculated from equations in *Principles of Engineering Economic Analysis.*[6]

$$A = \frac{1-[(1+j)^{20}(1+i)^{-20}]}{(i-j)} \quad \text{exponential series, present worth factor}$$

$$B = \frac{i(1+i)^{20}}{(1+i)^{20}-1} \quad \text{present to annual worth factor}$$

Note: the exponent 20 is the life cycle of the cash flow.

Energy cost of heat lost through roof insulation:

$$P_h = \frac{24F_hH}{RB_hE_h}(A)(B) = \frac{\left(\frac{\$}{\text{unit}}\right)\left(\frac{°\text{F} \times \text{days}}{\text{year}}\right)\left(\frac{24 \text{ hours}}{\text{day}}\right)}{\left(\frac{\text{hours} \times \text{ft}^2 \times °\text{F}}{\text{Btu}}\right)\left(\frac{\text{Btu}}{\text{unit}}\right)} = \frac{\$}{\text{ft}^2 \times \text{year}}$$

Energy cost for cooling caused by heat gain through roof insulation:

$$P_c = \frac{24F_cC}{RB_cE_c}(A)(B) = \frac{\$}{\text{ft}^2 \times \text{year}}$$

Insulation costs, P_I including materials, labor, and any added costs expected due to roof system changes caused by adding insulation.

$$P_I = JR(B) = \frac{\left(\frac{\$}{\text{ft}^2}\right)}{\left(\frac{\text{hr} \times \text{ft}^2 \times °\text{F}}{\text{Btu}}\right)} \times \frac{\text{hr} \times \text{ft}^2 \times °\text{F}}{\text{Btu}} = \frac{\$}{\text{ft}^2 \times \text{year}}$$

COST OPTIMIZATION

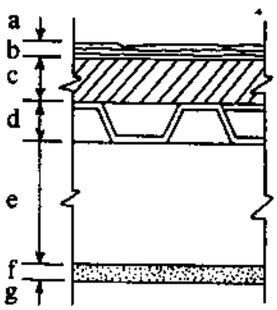
The total cost of energy lost through a roof can be expressed as the sum of initial insulation costs and costs of fuel due to energy losses or gains through installed insulation (*Figure 2*). Insulation costs are directly proportional to R and therefore plot as a straight line. Fuel costs are inversely proportional to R and therefore plot as a decreasing exponential curve. Adding the two plots (I.C. + F.C.) produces a total cost curve (T.C.).

The minimum cost point on the total cost curve corresponds to the optimum value of R to use for insulation. The minimum cost is obtained by taking the first derivative of the total cost equation with respect to R and setting the differential equation equal to zero.

The options for the two-layer system are limited to a 3-inch thickness due to design parameters. From local contractors, the estimated cost for this insulation system is $0.08 per R unit.

$$R_{opt} = 4.03\sqrt{\frac{1}{.08}} = 14.25$$

To see if this meets the Massachusetts Code[9] of overall U=0.07 or overall R=14.3, we take a sample section through the roof structure.



	R thermal resistance*
a. outside air film (15-mph wind)	0.17
b. 3/8-inch built-up roof membrane	0.33
c. insulation	14.25
d. steel deck	0
e. air space	0.99
f. 5/8-inch ceiling tile	1.56
g. inside air film	0.61
R_T	17.91

*Values from NRCA Energy Manual[2]

For this example $R_T = 17.91$. This R-value yields an overall U value of 1/17.91 (=0.056), which is less than 0.07. Therefore, the value for optimum R will also cause overal R value which exceeds state code requirements.

Had the minimum amount of insulation been used to meet the code, overall R=14.3, by subtracting the other R values of the system we find a minimum R value of 10.64. The economic penalty for merely meeting the state code would be:

$$p = \frac{50}{\left(\frac{10.64}{14.25}\right)} + 50\left(\frac{10.64}{14.25}\right) - 100 = 4.3\%$$

Note that in some cases the insulation's optimum R may yield an overal R value less than the code requires. Increasing R to meet code causes economic loss in such a case.

REFERENCES

1 Yamamoto, G.S. (Ed.), "Energy Conservation in New Building Design," Standard 90-75, American Society of Heating, Refrigerating, and Air Conditioning Engineers. New York, 1975.

2 *NRCA Energy Manual*, National Roofing Contractors Association, Oak Park, Illinois, September 1977.

3 Cash, C.G. "Optimization of the Thermal Resistance of Roof Insulation in Reroofing," Proceedings of the 1977 Symposium on Roofing Technology, National Bureau of Standards and the National Roofing Contractors Association, Paper No. 24, pp. 208-214.

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5 Amspec Inc., publication 7.14 Ams., Building Insulation, Board Insulation, Sweet's Catalogue, Section 7p, September 1975.

6 Agee, M.H., Case, K.E., White, J.A., *Principles of Engineering Economic Analysis*, New York: John Wiley & Sons, 1977.

7 Gumpertz, W.H., "Performance Considerations for Thermally Efficient Roofing Systems," Fiftieth Roofing Technology Conference, Gaithersburg, Maryland, April 1979; Proceedings, published by the National Roofing Contractors Association, Oak Park, Illinois, April 1979.

8 "Climatography of the United States," U.S. Department of Commerce, National Oceanic and Atmospheric Administration, Environmental Data Service, Asheville, North Carolina, No. 81 (by state).

9 "Building Code Provisions for Energy Conservation," Massachusetts State Building Code, Art. 22, 1977.

10 Owens-Corning Fiberglas, publication 7.15/OW, p. 12, Building Insulation. Sweet's Catalogue 7, September 1977.

EFFICIENCIES OF HEATING SYSTEMS*	
Hand-Fired Anthracite Furnace	60-75%
Hand-Fired Bituminous	50-65%
Stoker-Fired Coal	60-75%
Oil and Gas Fired	70-80%
Gas Designed Unit	75-80%
Gas Converted Unit	60-80%
Oil Designed Unit	65-80%
Oil Converted Unit	60-80%
Direct Electric	100%
Steam	100%
Heat Pump	varies up to 400%

Table 1

**Heating System efficiencies will vary. The specific values should be obtained from the building owner. If the specific values are not available, these values may be used as estimates.*

HEATING VALUE OF FUELS*	
Anthracite Coal	12,910 Btu/lb
Bituminous Coal	9,150 Btu/lb
#2 Fuel Oil	136,000 - 144,000 Btu/Gal.
#5 Fuel Oil	146,000 - 155,000 Btu/Gal.
#6 Fuel Oil	150,000 - 156,000 Btu/Gal.
Natural Gas	1,000 Btu/Cu. Ft.
LPG	91,690 Btu/Gal.
Steam	1,000 Btu/lb
Electricity	3,413 Btu kwh

Table 2

**Fuel heating values will vary. Specific values can be obtained from suppliers or building owners. If specific values are not available these values may be used as estimates.*

ANNUAL DEGREE DAYS

Location	Heating	Cooling
Charlotte Amalie	0	5427
Miami, Fl	206	4038
Orlando, Fl	704	3447
Savannah, GA	1952	2317
Augusta, GA	2547	1995
Atlanta, GA	3095	1598
Williamsburg, VA	3671	1345
Richmond, VA	3939	1353
Washington, DC	4211	1415
Baltimore, MD	4729	1108
Elizabeth, NJ	5017	953
Bridgeport, CT	5461	735
Port Jervis, NY	6087	622
Amherst, MA*	6576	511
Binghamton, NY	7285	369
Hanover, NH	7680	327
Massena, NY	8237	343
Old Town, ME	8648	209
Presque Isle, ME	9135	163
Caribou, ME	9632	128

Table 3 *used in sample calculation

$$P_o = \begin{cases} P_1 \left[\dfrac{1 - (1+j)^{20}(1+i)^{-20}}{i - j} \right], & i \neq j \\ \dfrac{P_1 (20)}{1+i}, & i = j \end{cases}$$

where P_1 = current yearly heating & cooling cost

P_o = present worth value of 20 year heating & cooling costs

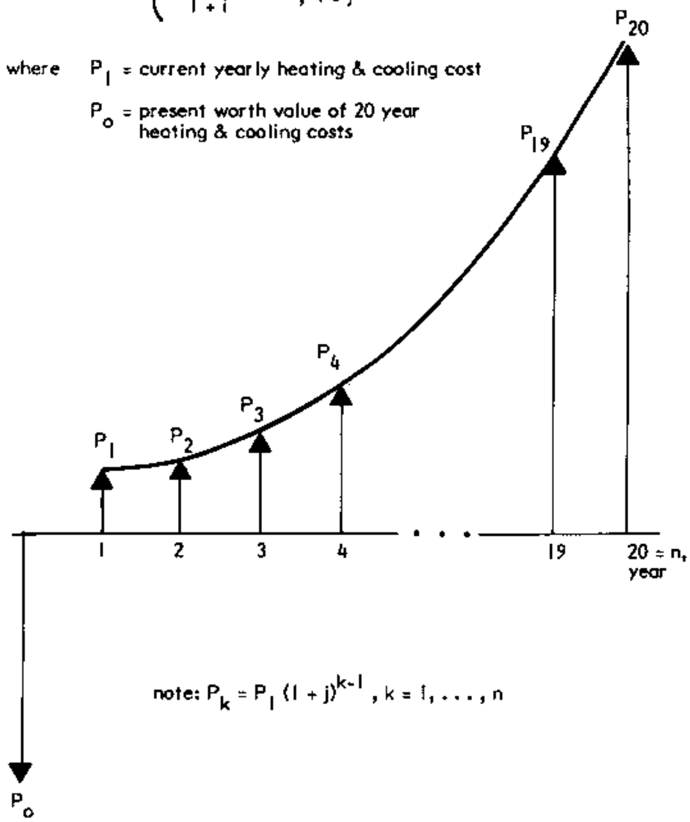

Figure 1 *Exponential cash flow diagram for fuel costs*

Step Three—
Initial Equipment Savings

For new buildings, determine the projected cost reductions for initial heating and cooling equipment by using the dollar multipliers supplied in Table. Multiply the summer "U" improvement by the dollar multiplier for cooling and the winter "U" value by the dollar multiplier for heating to calculate the estimated equipment savings per 1,000 sq. ft. of roof area. Add heating and cooling equipment reductions for total initial mechanical cost reductions per 1,000 sq. ft. of project roof area.

Equipment	Zone	Type Deck	$ Multipliers
Heating (gas)	I	All types	32.00
	II	All types	28.00
	III	All types	20.00
Heating (oil)	I	All types	32.00
	II	All types	28.00
	III	All types	20.00
Heating (electric)	I	All types	19.20
	II	All types	16.80
	III	All types	12.00
Cooling (electric)	All zones	Structural concrete 4"	57.80
		Structural concrete 6"	51.10
		Gypsum 2"	60.00
		Light weight concrete 2"	60.00
		Tectum Plank	64.40
		Wood	64.40
		Metal	68.90

Table 4 *Equipment cost reduction (dollars per "U" improvement/1000 square feet)*

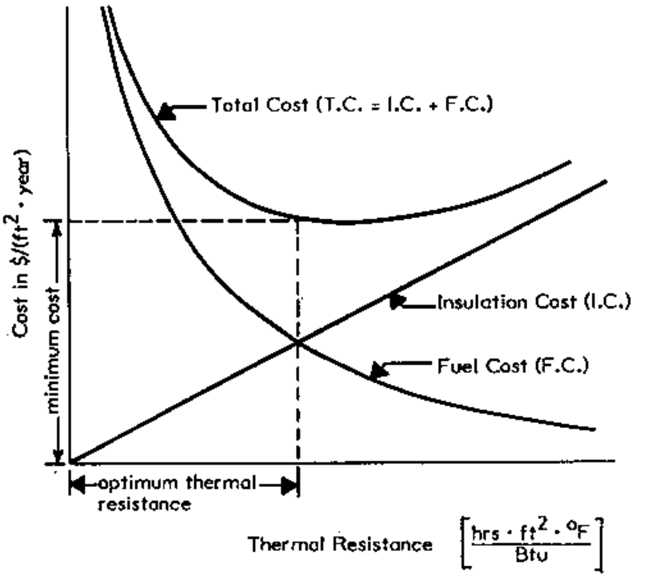

Figure 2 Determination of optimum thermal resistance.

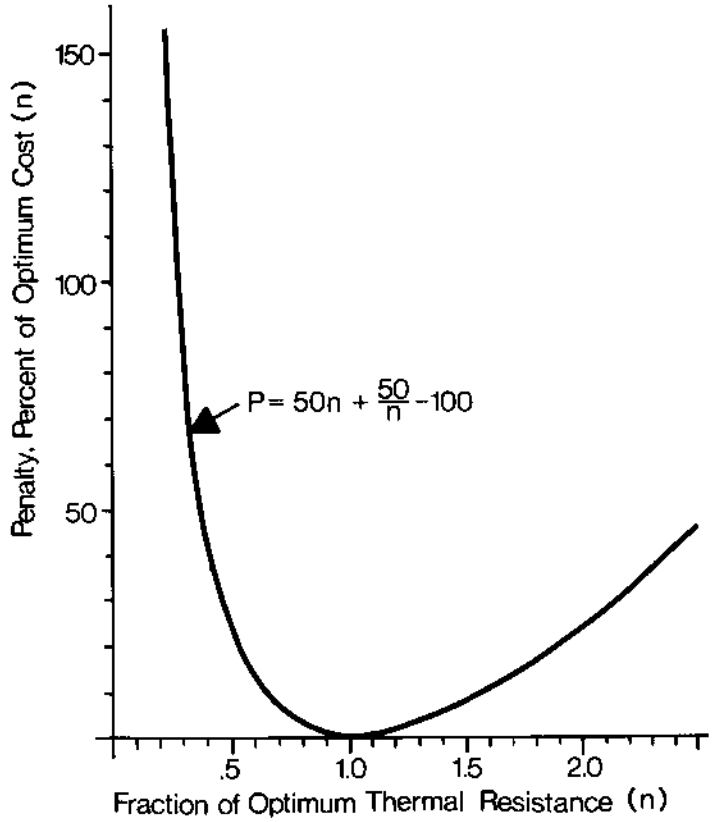

Figure 3 Penalty, % of optimum cost for not using optimum thermal resistance