STRESS-STRAIN CHARACTERISTICS FOR EPDM, CSPE AND PVC FOR THE DEVELOPMENT OF STRESSES IN MEMBRANES UTILIZED AS SINGLE-PLY ROOF SYSTEMS

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The work shows the development of stress vs. strain characteristics for EPDM, reinforced CSPE (also known as Hypalon) and reinforced PVC single-ply roofing membranes. Utilizing conventional definitions for stress and strain, corresponding stress and strain measurements are developed from standard tensile testing techniques. The resultant conventional stress-strain curves are shown to be curvilinear. However, when represented in a classic log-log relationship, the stress-strain relationships are found to follow the classic power equation. The work is further expanded to examine the strains in mutually perpendicular directions. The resultant changes in thickness for the non-reinforced membranes are shown to be of considerable magnitude and in the reinforced membranes are shown not to be significant. The work developed identifies for the first time how stress-strain criteria for the nonlinear roofing membranes can be incorporated into the design parameters for single-ply roof systems. Finally the work demonstrates that the ultimate stress criteria, ultimate strain criteria, and energy to rupture criteria for the EPDM, reinforced CSPE, and PVC are significantly different, and can play an important roll in the design phase when utilized as single-ply roofing membranes. The work also includes the influence of temperature vs. time on stress-strain characteristics of EPDM. The results demonstrate that the stress-strain characteristics are altered substantially by temperature on EPDM. The work identifies for the first time the complete stress-strain characteristics for EPDM, reinforced CSPE and PVC membranes that can be utilized for designing single-ply roof systems.

SYMBOLS

A	Nominal cross sectional area (in.^{2}) (mm^2)
A_T	True cross sectional area (in.^{2}) (mm^2)
$A(m,n)$	Constants and exponents
b	Width of membrane strip (in.) (mm)
C,D	Constants
C_p	Coefficient of pressure (dimensionless)
F	Force in membrane (pounds) (newtons)
g	Gravitational acceleration (32.2 fps^2) (9.80 m/s^2)
h	Maximum height of deformed membrane (in.) (mm)
H	Building height (ft.) (m)
L	Half width of strap centerlines (in.) (mm)
L_o	Undeformed length of membrane (in.) (mm)
L_f	Deformed length of membrane (in.) (mm)
M	Energy to rupture (in.-lbs.) (Joule)
P	Differential pressure (psi and psf) (pascals)
r	Radius of curvature (in.) (mm)
t	Initial thickness of membrane (in. and mils) (mm)
t_T	Thickness of deformed membrane (in. and mils) (mm)
v_o	Basic wind speed (mph and fps) (m/s^2)
v_R	Rooftop wind speed (mph and fps) (m/s^2)
x,y,z	Coordinate directions
ϵ	Strain (in./in.) (mm/mm)
γ	Specific weight of air (pcf) (N/m^3)
σ	Stress (psi) (pascals)
σ_T	True stress (psi) (pascals)
θ	Deflection angle (radians)
$\oplus$	Domain separator for equations

CONVERSION FACTORS

1 in.	= 25.4mm
1 in.^{2}	= 645.2mm^2
1 ft.	= 3.048×10^{-1}m
1 psi	= 6.895×10^3 Pa
1 lb	= 4.448 N
1 mph	= 0.447 m/s^2
1 in-lb	= 6.146×10^{-2} Joule
1 pcf	= 157 N/m^3

INTRODUCTION

Single-ply roofs have become an increasingly common roofing system and today occupy about 50 percent of the roofing market in the United States. EPDM (ethylene-propylene-diene terpolymer), reinforced CSPE PVC (polyvinyl chloride) are common materials utilized as single-ply membranes. Stresses are developed in all roof systems whether adhered, ballasted or mechanically fastened. As an illustration, this paper uses a mechanically-fastened system utilizing anchor bars or straps subjected to wind loading as shown in Figure 1. The seams are bonded together by either a chemical bond using an adhesive spread at the seams, or a heat weld using a heating element to soften the membrane at the seam and pressing together.

Differential pressure caused by wind and other internal effects can make the membrane balloon up, straining the membrane. The stress-strain characteristics of the materials will assist in the accurate prediction of the resulting stresses and the height of the deformation of the membrane.

STRESS MODEL

Observation has shown that a single-ply membrane will develop into a balloon under the effects of a differential pressure created by wind or other external effects. The shape approximates a circular arc as developed by experimental procedures at Northeastern University through 1300 data points as shown in Figure 2.

By taking the differential strip, the resultant force relationship shown in Figure 3 can be developed. Since the membrane is thin, it is reasonable to assume uniform stress over the cross section as is the practice in other classic thin-walled pressure vessel theory. Therefore the force in the membrane is

$$F = \sigma \, t \, \Delta y. \tag{1}$$

Summation of forces in the z direction yields

$$2 \, p \, L \, \Delta y \cdot 2 \, \sigma \, t \, \Delta y \sin\theta = 0 \tag{2}$$

and solving for the membrane stress, σ, results in

$$\sigma = \frac{PL}{t \sin\theta}. \tag{3}$$

The geometry of the balloon based upon laboratory observations shown in Figure 2 is shown in Figure 4, and therefore is based on the properties of a circular arc.

It is easily shown that the radius r can be written as

$$r = \frac{L^2 + h^2}{2h}. \tag{4}$$

Since $\tan \theta/2 = h/L$

$$\theta = 2\tan^{-1}\left(\frac{h}{L}\right) \tag{5}$$

where θ is in radians. Also noting that $\sin\theta = L/r$ and combining with Equation 4 yields

$$\sin\theta = \frac{2hL}{L^2 + h^2}. \tag{6}$$

Substituting Equation 6 into Equation 3 yields

$$\sigma = \frac{P(L^2 + h^2)}{2th}. \tag{7}$$

This approach was first developed in Reference 1.

STRAIN MODEL

Using a simple one dimensional strain model the strain may be written as

$$\epsilon = \frac{\Delta L}{L_o} \tag{8}$$

or

$$\epsilon = \frac{L_f - L_o}{L_o}. \tag{9}$$

Recalling Equations 4 and 5, L_f can be written as

$$L_f = 2\left[\frac{L^2 + h^2}{2h}\right]\left[2\tan^{-1}\left(\frac{h}{L}\right)\right]. \tag{10}$$

Substituting Equation 10 into Equation 9, the strain becomes

$$\epsilon = \frac{2\left[\frac{L^2 + h^2}{h}\right]\tan^{-1}\left(\frac{h}{L}\right) - L}{L}. \tag{11}$$

or

$$\epsilon = \frac{2L}{h}\left[1 + \left(\frac{h}{L}\right)^2\right]\tan^{-1}\left(\frac{h}{L}\right) - 1. \tag{12A}$$

Using the following series expansion for $\tan^{-1}$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} \quad -1 \leq x \leq 1. \tag{12B}$$

Equation 12A can be written as

$$\epsilon = \left[\frac{2L}{h}\left[1 + \left(\frac{h}{L}\right)^2\right]\left[\frac{h}{L} - \frac{1}{3}\left(\frac{h}{L}\right)^3 + \frac{1}{5}\left(\frac{h}{L}\right)^5 \cdots\right]\right] - 1. \tag{13}$$

Expanding and combining terms results in

$$\epsilon = \frac{2}{3}\left(\frac{h}{L}\right)^2 - \frac{2}{15}\left(\frac{h}{L}\right)^4 + \frac{2}{35}\left(\frac{h}{L}\right)^6 \cdots \tag{14}$$

or

$$\epsilon = 2\sum_{n=0}^{\infty}\left[\frac{(-1)^n}{(2n+1)(2n+3)}\left(\frac{h}{L}\right)^{2n+2}\right] \quad -1 \leq \frac{h}{L} \leq 1. \tag{15}$$

When the strain relationship developed in Equations 14 and 15 is represented in a log-log plot of strain (ϵ) vs. the balloon height ratio (h/L), a linear relationship is developed as shown in Figure 5.

Such a linear relationship in a log-log plot can be represented by the power equation of the form

$$y = Cx^D \tag{16A}$$

where the constants C and D are evaluated by conventional computational methods. A simpler form of strain vs. balloon height ratio (h/L) can be derived from Figure 5 as follows.

$$\epsilon = 0.602\left(\frac{h}{L}\right)^{1.975}. \tag{16B}$$

Thus, Equation 16 may be utilized as a simpler form of expressing the resultant strain of single-ply membranes from the balloon height ratio for a single-ply roof system strapped by anchor bars. The previous equations are for systems strapped by anchor bars. Similar relationships for spot-fastened systems can be developed but are beyond the impact of this paper at this stage.

STRESS-STRAIN RELATIONSHIP

Observations show that the stress-strain plot of membrane materials does not follow a straight line relationship.[1] It does, however, follow a power relationship in the form of

$$\sigma = C\epsilon^D \tag{17}$$

where C and D are empirical quantities developed from tensile testing procedures, and will be shown to follow Equation 17 later in this paper. The C and D quantities noted in Equation 17 are developed from the mathematics associated with the derivation of the power equation form from the plots of data in the log-log format.

Substituting Equation 12 into Equation 17 yields

$$\sigma = C\left(\frac{L}{h}\left[1+\left(\frac{h}{L}\right)^{2}\right]\tan^{-1}\left(\frac{h}{L}\right)-1\right)^{D} \tag{18}$$

and substituting Equation 7 into Equation 18, and solving for P gives

$$P = \frac{2tC}{h\left[1+\left(\frac{L}{h}\right)^{2}\right]}\left(\frac{L}{h}\left[1+\left(\frac{h}{L}\right)^{2}\right]\tan^{-1}\left(\frac{h}{L}\right)-1\right)^{D}. \tag{19A}$$

Substituting Equation 16 into Equation 17 and equating the result to Equation 7 yields a simpler relationship for the differential pressure p, or

$$p = \frac{2Ct(0.602)^{D}(h)^{(1.975D+1)}}{(L^{2}+h^{2})L^{(1.975D)}}. \tag{19B}$$

Therefore, it is incumbent in the stress analysis and/or design that the membrane parameters C and D be identified from the power Equation development or the design or stress cannot be properly documented.

LABORATORY TESTING

The membrane parameters C and D were evaluated in a tensile test program at Northeastern University for each type of membrane being considered. The shape of the test specimens are shown in Figure 6.

For those materials containing reinforcing, it was important to align the scrim, or reinforcing fibers, parallel to the longitudinal axis to avoid fiber runout which significantly effected the test results. The undeformed length (L_o) was measured at two gauge marks. The applied force (F) elongated the specimen yielding a difference in length (ΔL). The strain is computed by using Equation 8 and the nominal stress by

$$\sigma = \frac{P}{A}. \tag{20}$$

The resulting graph of stress vs. strain is not a straight line. However, if the log stress vs. log strain is plotted, the resultant curve does follow a straight line, indicating that stress is related to strain by the power equation of Equation 17. During the elongation of the specimen, the thickness and width are also observed to be decreasing. True stress, defined to be the applied force divided by the actual cross sectional area, is given by

$$\sigma_T = \frac{P}{A_T} = \frac{P}{b_T t_T}. \tag{21}$$

STRESS-STRAIN CHARACTERISTICS OF EPDM

It was found in Reference 2 that the stress-strain plot of EPDM, a non-reinforced membrane, follows the power relationship of Equation 17 as shown in Figure 7. The values of constants C and D are shown in Table 1.

Substituting the average values of C and D into Equation 17 yields

$$\sigma = 600\ \epsilon^{0.791} \tag{22}$$

when the experimental stress and strain is as developed from the testing model shown in Figure 7.

This paper also investigates the effects of change in thickness on the stress-strain relationship. Tests were made measuring the change in thickness of specimens while loading to failure as shown in the stress model in Figure 6. Using Equation 21, the true stress vs. strain is shown in Figure 8. The relationship for true stress is then determined to be

$$\sigma_T = 595\ \epsilon^{1.720}. \tag{23}$$

The representations of nominal stress vs. true stress is shown in Figure 9. The relationship can be written as

$$\sigma_T = 0.0966\ \sigma_N^{1.559}. \tag{24}$$

Thickness vs. nominal stress as shown in Figures 10 is related by

$$\Delta t = 1.45\ \sigma_N^{0.528}. \tag{25}$$

It should be noted that the width of the test specimens was 0.5 in., and they changed in width and thickness. The behavior of very wide membranes would be different. However, Equation 25 represents an upper bound of the true stress.

In various locations, an ambient temperature of 95°F (35°C) can result in a roof temperature as high as 190°F (88°C), which can occur many times over the life of the roof. This influence has been demonstrated by actual field measurements developed in References 3 and 4. In order to simulate this cumulative effect of temperature, specimens were cured at a temperature of 240°F (116°C) for 0,2,4,6,8 and 10 weeks, and then their stress-strain properties were tested. The 240°F (116°C) curing temperature was also selected to coincide with industry temperatures for heat aging in accordance with ASTM D 573 as noted in industrial specifications noted in Reference 5. The results on the effect onr constants C and D are shown in Table 2.

The data developed in Table 2 is represented in Figure 11 and depicts ultimate stress vs. time and ultimate strain vs. time for EPDM when aged at 240°F (116°C) for a period of 10 weeks.

The authors have noted that both ultimate stress and ultimate strain decrease initially for the first two to three weeks and then increase lightly in the fourth week and then decrease and appear to reach a relatively constant value between the eighth and tenth weeks when cured at 240°F (116°C). The significant factor is that the ultimate strain reductions in the order of 57 percent and the apparent modulus (σ/ϵ) increases in the order of 75 percent. This data indicates that EPDM does undergo embrittlement in the aging process. Clearly the strain and stress at failure are severely influenced after aging.

STRESS-STRAIN CHARACTERISTICS OF REINFORCED CSPE

Tensile tests on reinforced CSPE were performed in accordance with ASTM D 638. Particular attention to the cut of the specimens was made. If run-out (reinforcing fibers not running continuously along the specimen) occurred then the specimen would fail prematurely. The results of the tests are shown in Figure 11.

The results of Figure 11 indicate that there are two portions of the curve. Since the average value of stress vs. strain closely follows a straight line on the log-log plot, it is possible to represent each section in the form of Equation 16.

$$\sigma = C\epsilon^{D}\Big]_{\epsilon_0}^{\epsilon_1} \oplus C'\epsilon^{D'}\Big]_{\epsilon_1}^{\epsilon_2} \qquad (26)$$

Determining the constants for each section yields

$$\sigma = 6600\ \epsilon^{0.545}\ \Big]_{0.0095}^{0.071} \oplus 17600\ \epsilon^{0.914}\ \Big]_{0.071}^{0.195}\ . \qquad (27)$$

The knee represents a transition point in the stress-strain behavior. To the left of the knee, the membrane and the reinforcing are acting compositely. The right side is where the membrane reinforcing is carrying the entire load and stress.

Time constraints did not allow for the study of effects of temperature over time.

STRESS-STRAIN CHARACTERISTICS OF REINFORCED PVC

The stress-strain characteristics of reinforced PVC were also tested in the same fashion as reinforced CSPE. The results are shown in Figure 12.

Like CSPE, the stress-strain relationship is specified by two domains which is given by Equation 28.

$$\sigma = 6220\epsilon^{0.760}\ \Big]_{0.000}^{0.085} \oplus 25800\ \epsilon^{1.33}\ \Big]_{0.090}^{0.200}. \qquad (28)$$

The total change in thickness observed near failure was four percent, which is negligible by engineering standards. This indicates that thickness changes in reinforced PVC essentially can be neglected. Time constraints did not allow for the study of effects of temperature over time.

IMPACT OF PRACTICAL USAGE OF STRESS-STRAIN CHARACTERISTICS

During the growth of the single-ply roof industry, the use of stress-strain characteristics in the area of design and analysis has been minimal. The development of the theory for systems strapped with anchor bars and the knowledge of how stress varies as a function of strain developed in this work offers a methodology for the first time for analysis and/or design of single-ply roof systems strapped with anchor bars by utilizing Equation 19 to determine the balloon height and inserting the value of the balloon height into Equation 7 to obtain the membrane stress.

This approach requires that the differential pressure acting on the membrane be established. This can be accomplished by establishing the rooftop wind speed. Reference 6 has demonstrated that all major codes and standards present data that allows the development of rooftop wind speeds (v_R) in the form of

$$V_R = AV_0^{m}H^{n}\ . \qquad (29)$$

This same data[6] also demonstrates that in the Uniform Building Code[7] for open country (Exposure C), the rooftop wind speed is given by

$$V_R = 0.854\ V_0^{0.985}H^{0.108}. \qquad (30)$$

For the condition of wind only, and no other positive pressure influences, the differential pressure acting on the membrane is given by

$$p = \frac{C_p\gamma V_R^2}{2g}. \qquad (31)$$

To illustrate the use of stress-strain characteristics, consider a structure on the coastline in open country that is 30 feet high. All standards indicate that a reasonable basic wind speed (V_0) is at least 90 mph in coastal areas. Substituting these values into Equation 29 yields a rooftop windspeed of 103.7 mph (152 fps). Utilizing this rooftop wind speed, a nominal value of the specific weight of air, gravitational acceleration as 32.2 fps^2 in Equation 30 with a coeffecient of pressure (C_p) of unity as found in the field of the roof, the differential pressure (p) (in psi) is identified as 0.195 psi.

In the field of the roof a common strap spacing is 72 in. Utilizing this spacing the dimension L becomes 72 in. With the pressure data just developed for L = 36 in., the balloon height, membrane stress, membrane strain and the change in thickness can be evaluated for EPDM, reinforced CSPE and reinforced PVC as shown in Table 3.

The data in Table 3 clearly indicates that balloon heights, membrane strain and change in thickness for EPDM are larger than their counterparts, reinforced CSPE and reinforced PVC, but the induced stress is substantially lower. The results in Table 3 are representative of comparative test results.

ENERGY TO RUPTURE

Classically, the energy to rupture (M) has been developed from the area under the stress-strain curve in the form of

$$M = \int \sigma\ d\epsilon \qquad (32)$$

Energy to rupture gives a measure of energy absorption to impact conditions and other dynamic loadings. The results of this interpretation for EPDM, reinforced CSPE, and reinforced PVC with the data presented herein are shown in Table 4.

The results clearly demonstrate that the energy to rupture in an EPDM membrane is substantially higher (by a factor of 10-15) than reinforced CSPE or reinforced PVC. The unreinforced membrane has larger deflections but will absorb greater amounts of energy. The reinforced membranes have smaller deflections and are limited to the amount of energy that can be absorbed.

CONCLUSIONS

The stress-strain characteristics of single-ply membranes can be represented by a simple form of a power, allowing the development of stress constants C and D is demonstrated by the fact that the constants are essential in the development of actual membrane stresses and strain in the field under actual wind conditions, and that these stresses and strains become predictable only when these membrane characteristics are known.

The energy absorptive capabilities are an indication of which membranes yield the higher balloon heights. The work clearly demonstrates that EPDM, due to its large energy absorptive capacity, yields lower membrane stresses in field conditions, but also yields substantially higher ballooning and substantive reduction in thickness when stressed in comparison to reinforced CSPE and reinforced PVC membranes.

The work clearly develops directions for methods of stress analysis and design in single-ply membranes of all types that is directly related to the proper development of membrane stress-strain characteristics.

The work demonstrates through sound testing protocol that stress can be related to changes in thickness of single-ply membranes and that EPDM does have significant thickness changes with stress, whereas the reinforced CSPE and PVC thickness changes are negligible. Thickness changes could have an adverse effect at fastener locations, causing slippage.

The work also demonstrates that EPDM's ultimate stress and strain are influenced by temperature by substantive reductions over an extended period of time in a similar fashion noted by Doherty and Schloss.[8]

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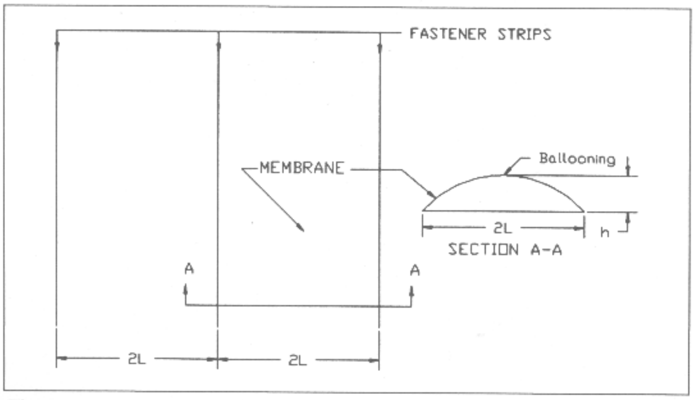
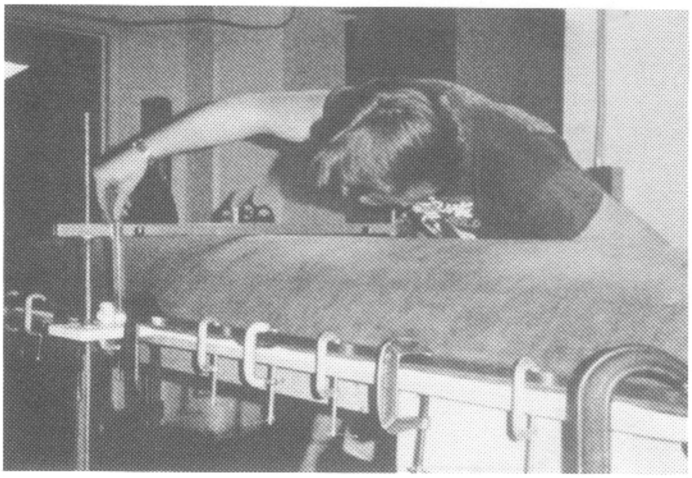


Figure 1



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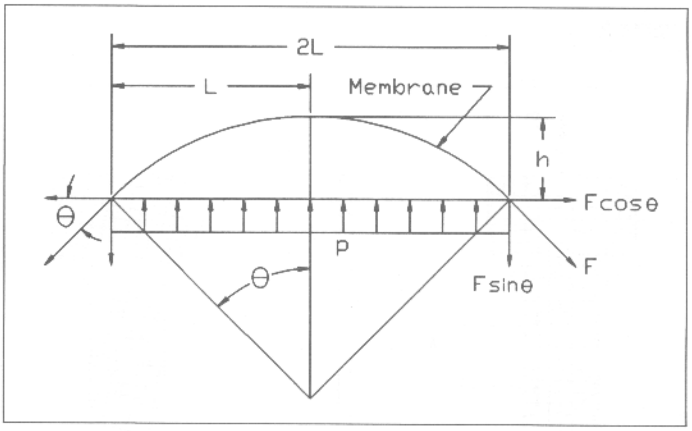

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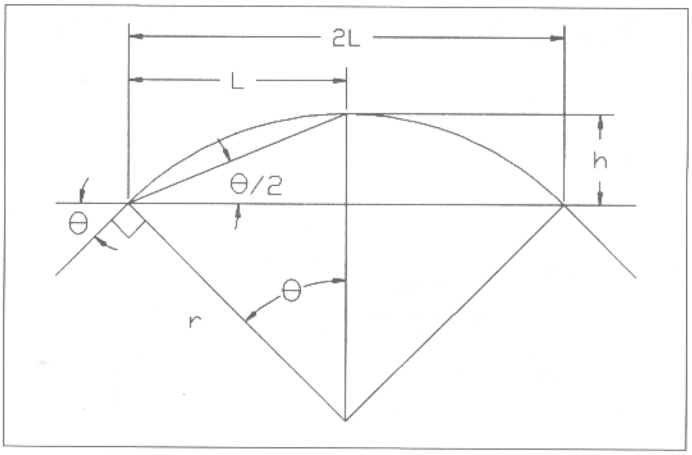

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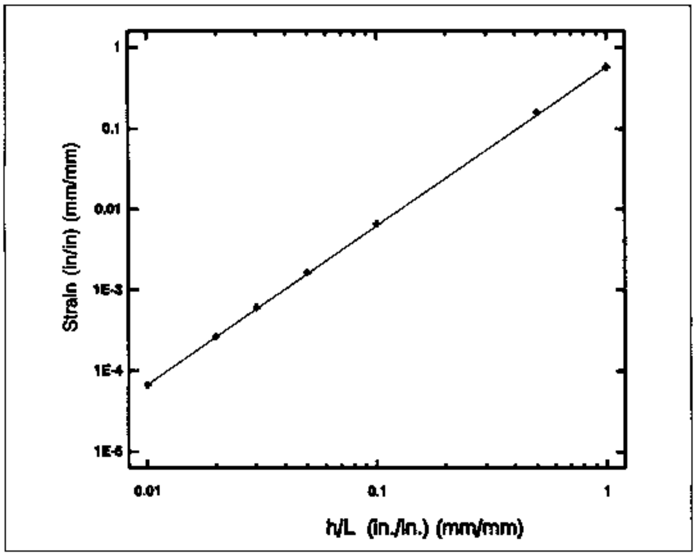

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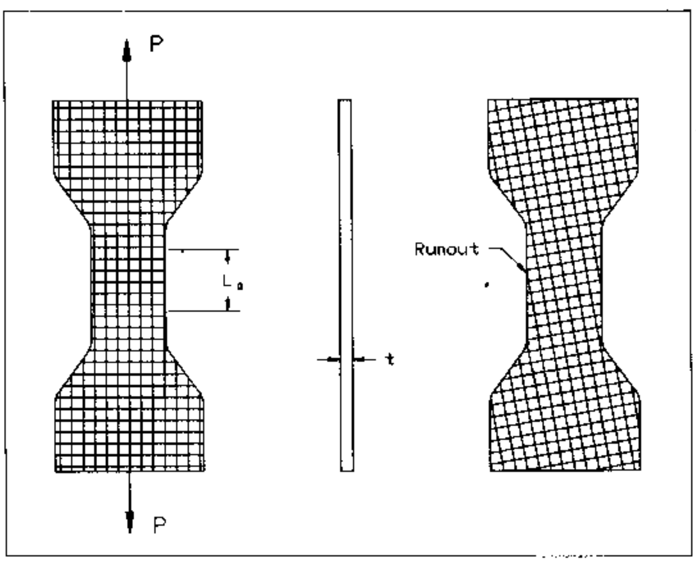

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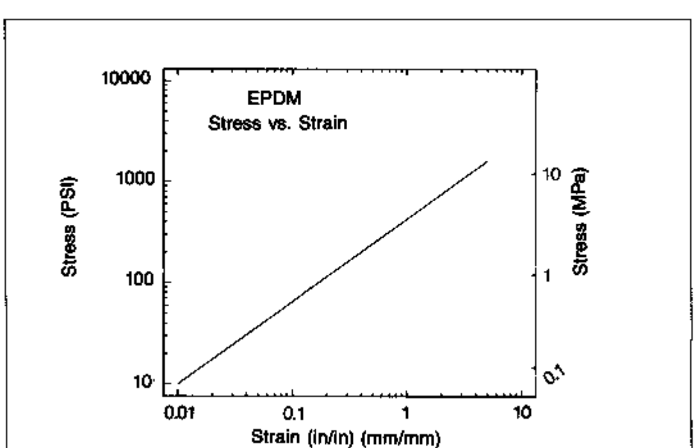

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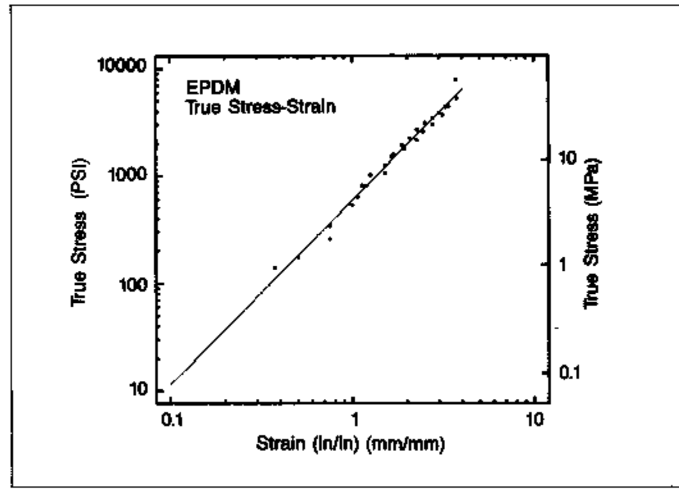

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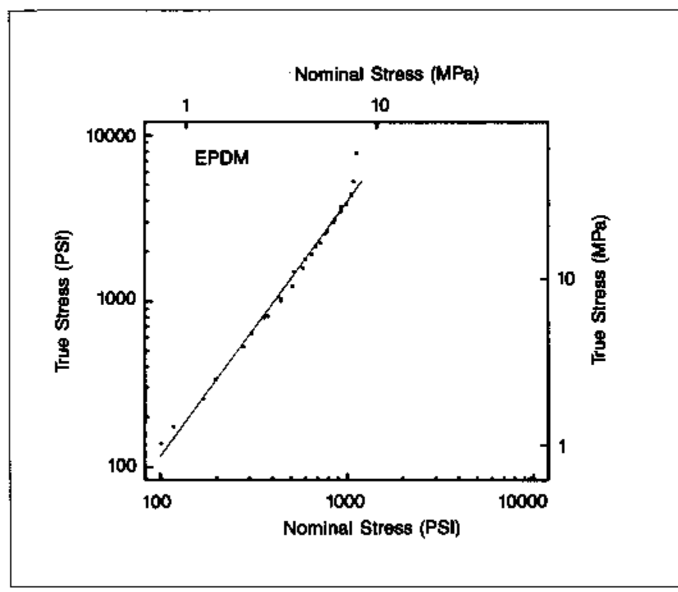

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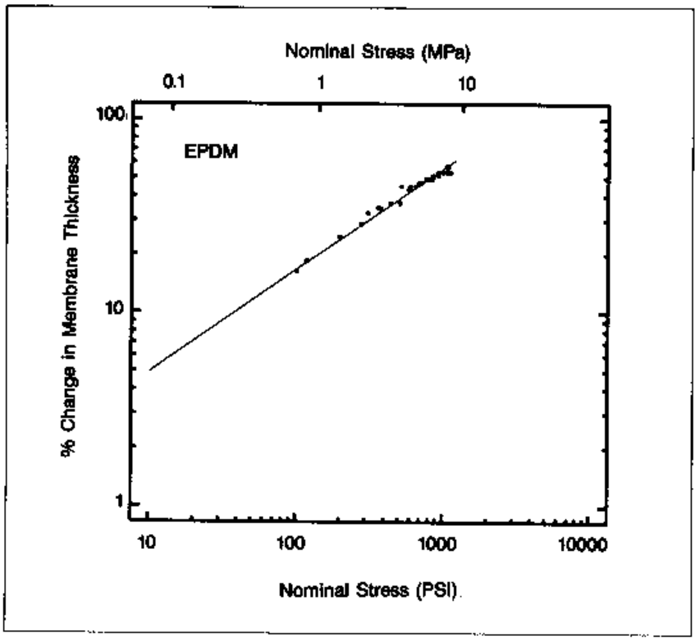

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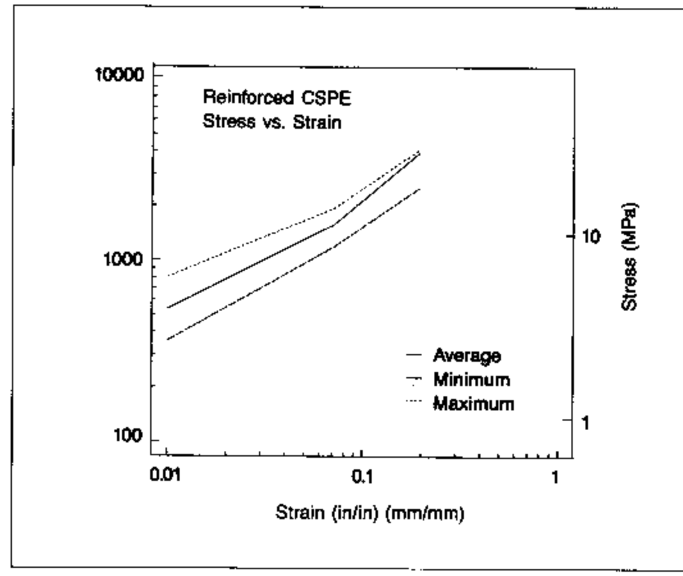

Figure 12

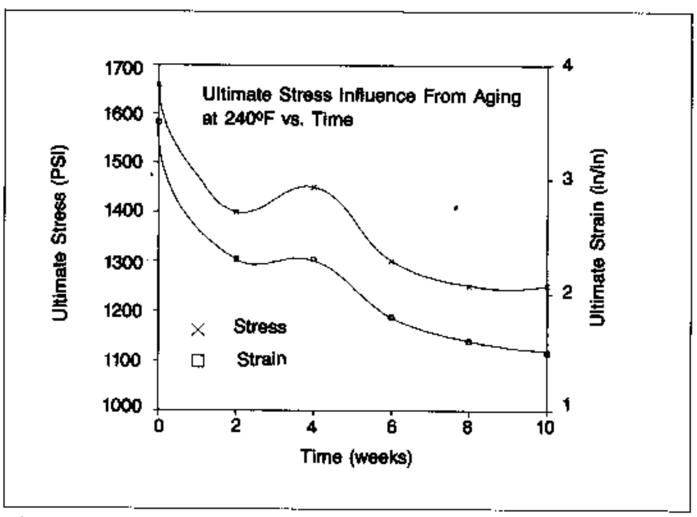

Figure 11

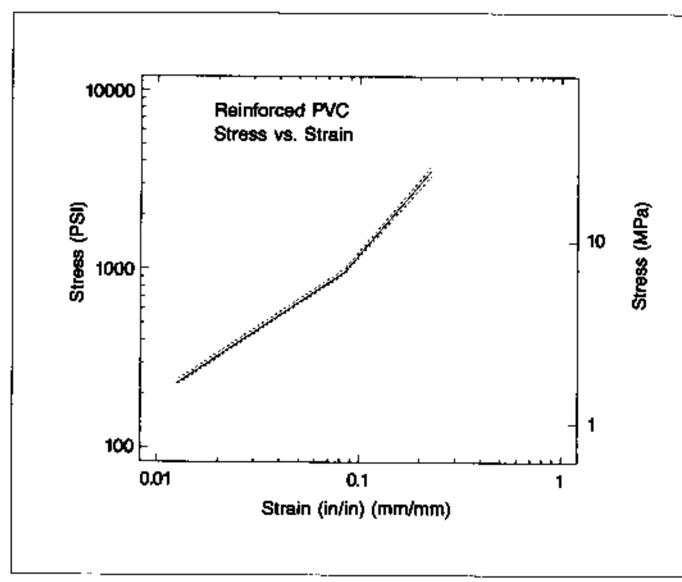

Figure 13

Constant	Lower	Upper	Median	Material in this Study
C	600	300	428	600
D	0.6	0.867	0.816	0.791

Table 1 *Stress-strain constants for EPDM.*

Week	C	D	$\epsilon_{ULTIMATE}$ in./in. and mm/mm	$\sigma_{ULTIMATE}$ psi (MPa)
0	600	0.791	3.5	1660 (11.4)
2	742	0.839	2.3	1400 (9.6)
4	731	0.747	2.3	1450 (10.0)
6	812	0.812	1.8	1300 (9.0)
8	812	0.813	1.6	1250 (8.6)
10	812	0.826	1.5	1250 (8.6)

Table 2 Influence of cumulative temperature on stress-strain criteria of EPDM.

Type of Membrane	Balloon Height in. (mm)	Membrane Stress psi (MPa)	Membrane Strain (%)	Change in Thickness (%)
EPDM	25.3 (643)	166 (1.14)	31.3	21.6
Reinforced CSPE	7.36 (187)	397 (2.74)	2.7	Negligible
Reinforced PVC	4.87 (124)	587 (4.05)	1.2	Negligible

Table 3 Critical membrane parameters (When v_0 = 90 mph [40.2 m/s²], v_R = 1 03.7 mph [46.4 m/s²], and L = 36 in. [914mm]).

Membrane	Energy (M) in.-lbs/in.³ (mJ/mm³)
EPDM	4380 (16.4)
Reinforced CSPE	416 (1.6)
Reinforced PVC	271 (1.0)

Table 4 Energy to rupture.